The Landscape of Lower Bounds for the Congest Model

Keren Censor-Hillel
Technion

TECHNION
Israel Institute
of Technology

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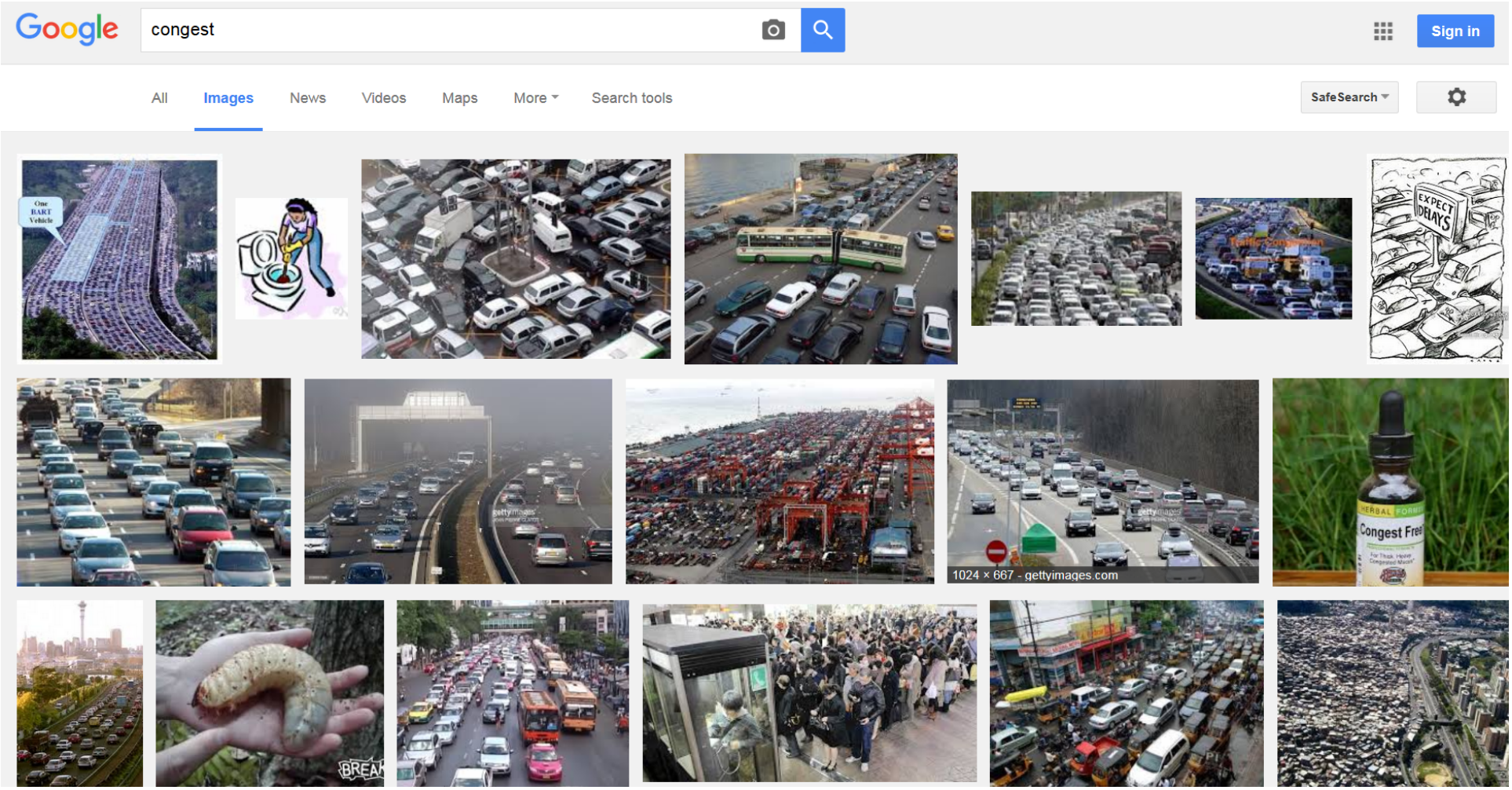
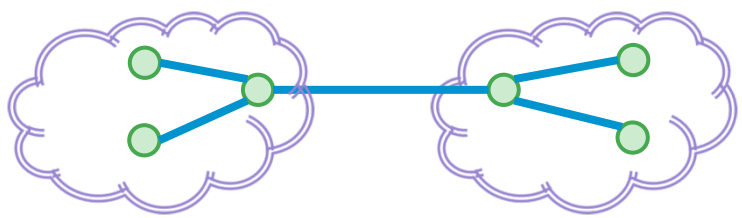
Some figures courtesy of Ami Paz

CONGESTION


Google
congest
Sign in
All
Images
News
Videos
Maps
More
Search tools
SafeSearch
One BART Vehicle
EXPECT DELAYS
1024 × 667 - gettyimages.com
Congest Free

CONGEST

- **n** nodes
- synchronous communication
- (**log n**)-bit messages

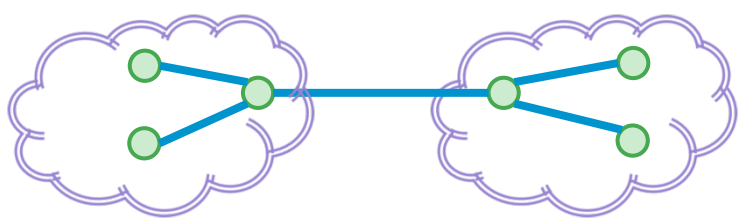


CONGEST

- **n** nodes
- synchronous communication
- (**log n**)-bit messages

Compare with LOCAL:

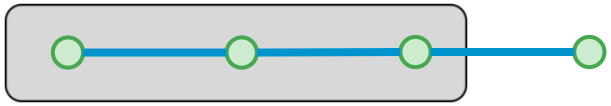
- no bound on message size



Global Problems - LOCAL

Computing the diameter **D**

Requires **Ω(D)** rounds



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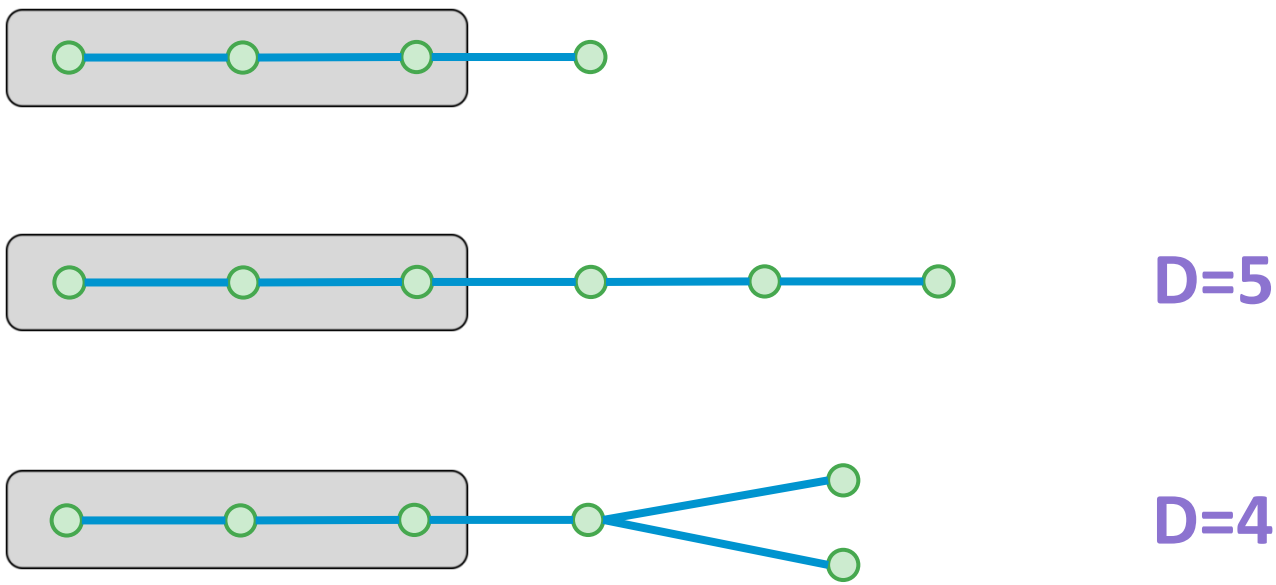
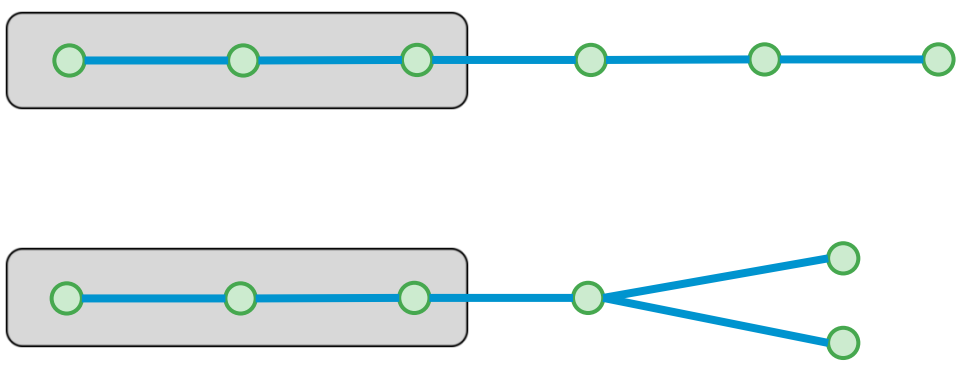


Global Problems - LOCAL

Computing the diameter **D**

Requires **Ω(D)** rounds

Easy to solve within **O(D)** rounds



D=5

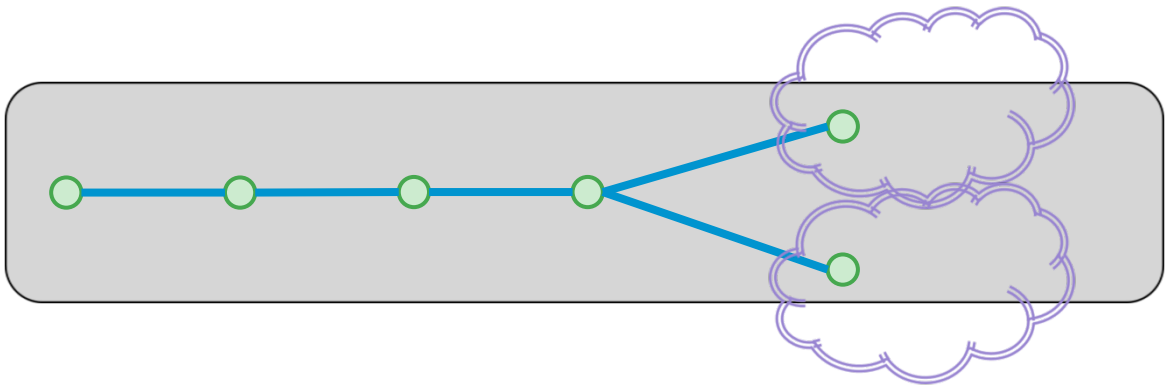
D=4

Global Problems – CONGEST

Distance is not the only aspect of communication

With **restricted bandwidth**:

need to design messages carefully



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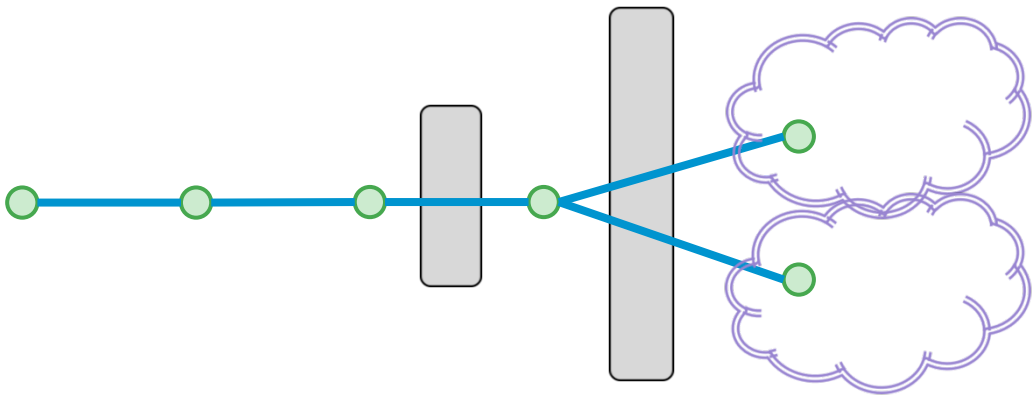

CONGEST – This Talk

- Flavor of some **known** lower bounds
- Glimpse into some **new** lower bounds
- Open problems – what we **need**
  - Where our current techniques fail

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Ready for a walk?

CONGEST – Computing **D**

Can be done in **O(n)** rounds

- in fact, even APSP

[Holzer, Wattenhofer 2012]
[Peleg, Roditty,Tal 2012]

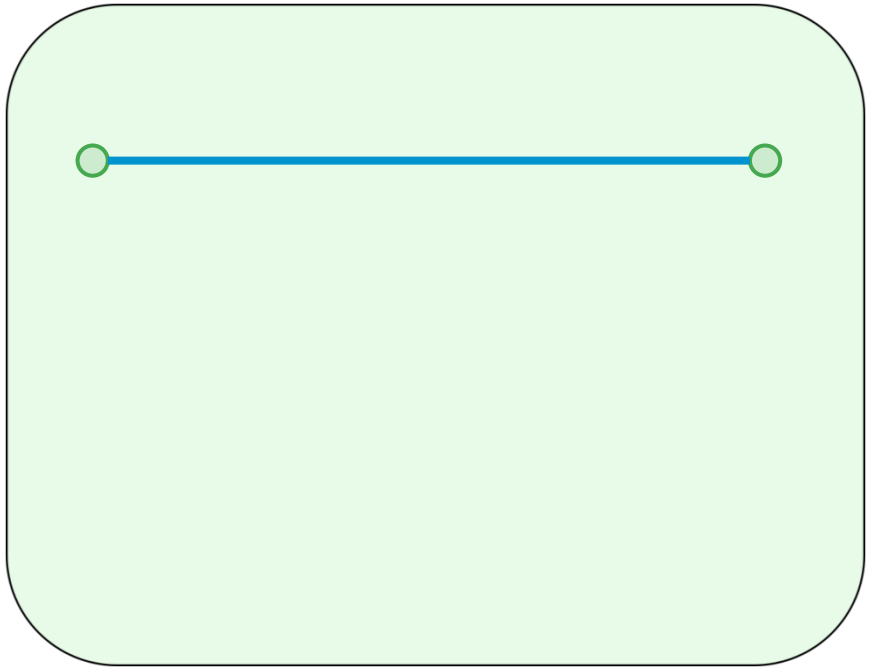
Lower bound of **Ω(n/logn)**

- even for small **D**

[Frischknecht, Holzer, Wattenhofer 2012]

Computing D

Ω(n/logn) lower bound for deciding **D=2** or **D=3**
[Frischknecht, Holzer, Wattenhofer 2012]



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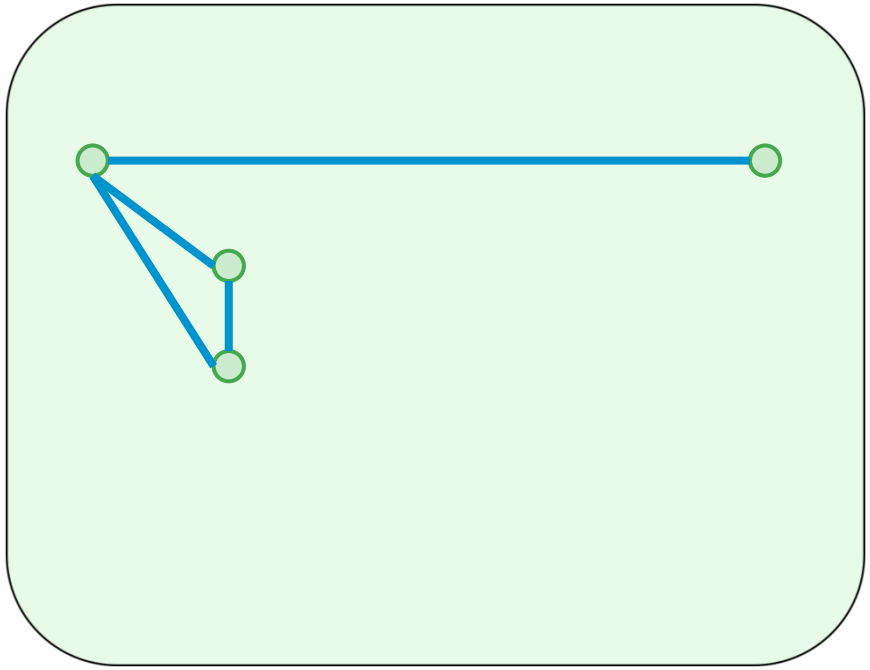

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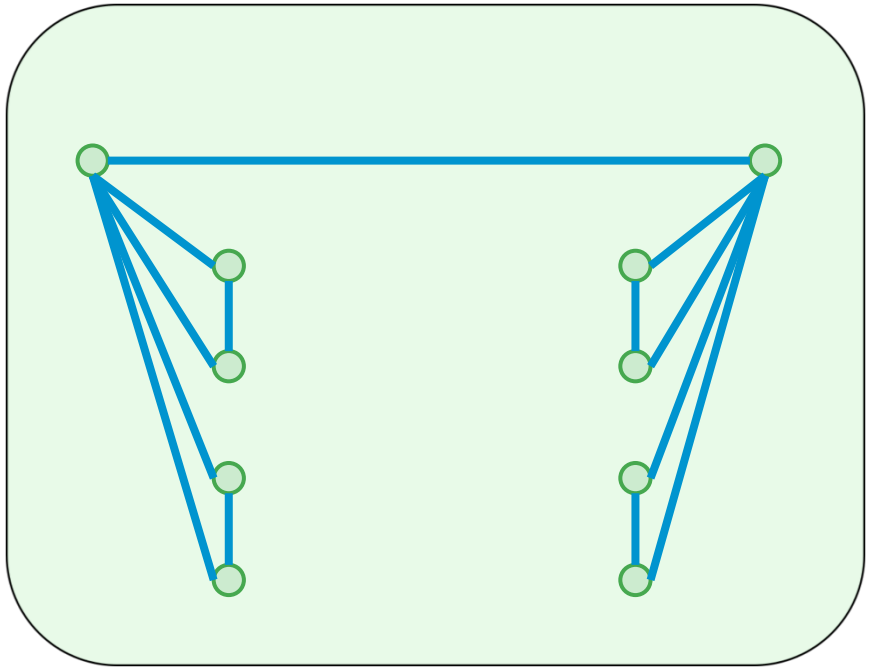

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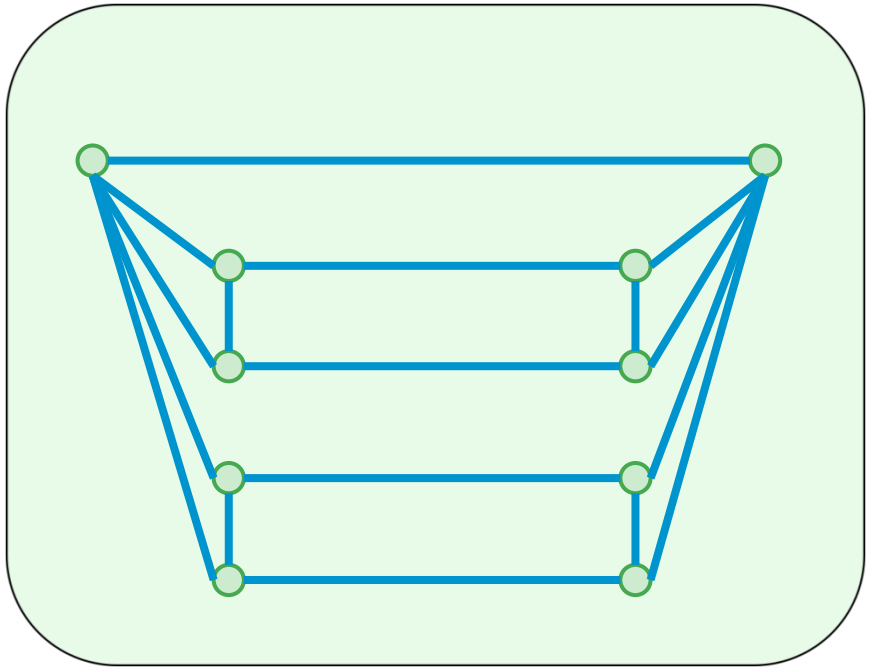

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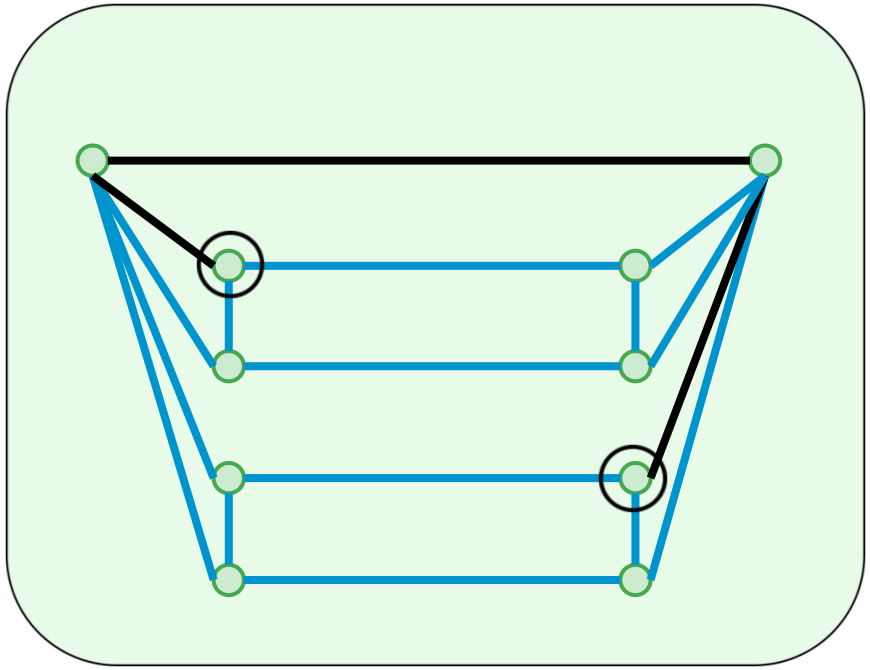

D=3

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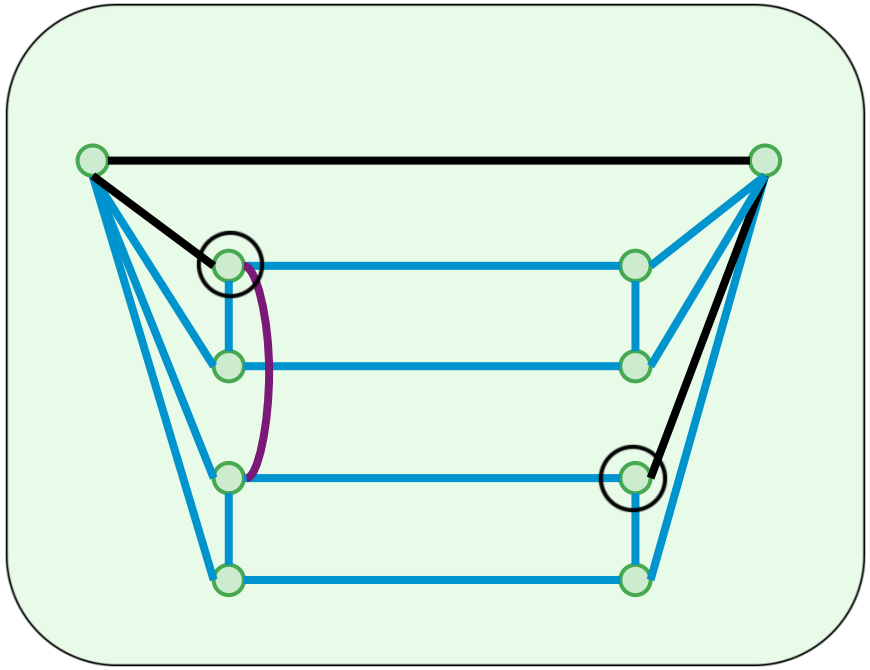

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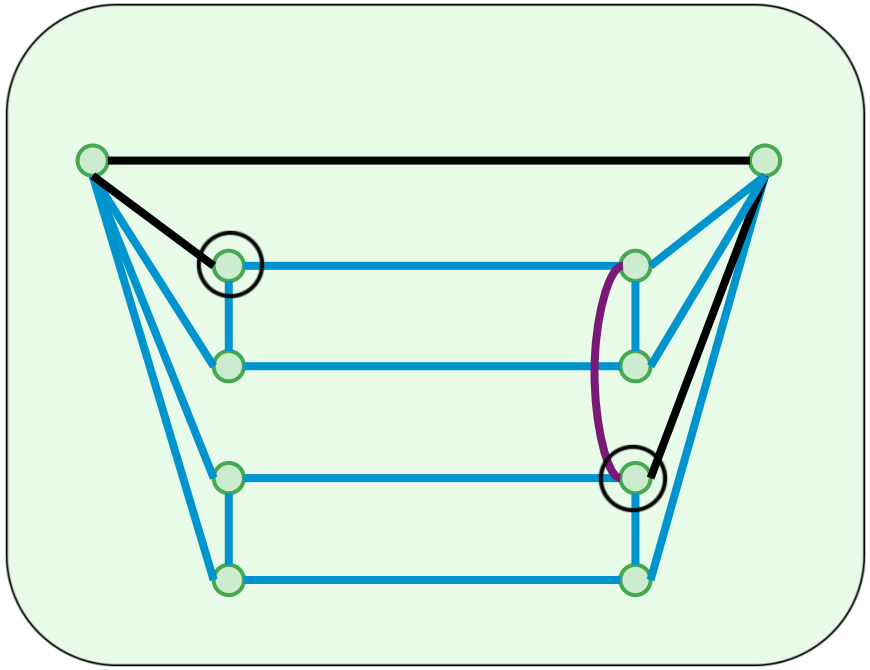

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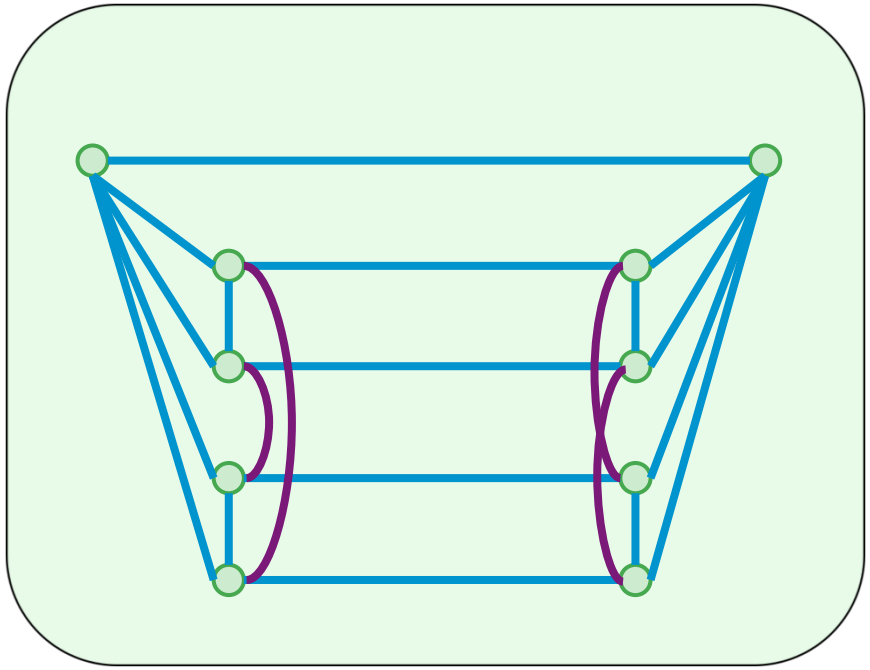

D=3

D=2

Computing D

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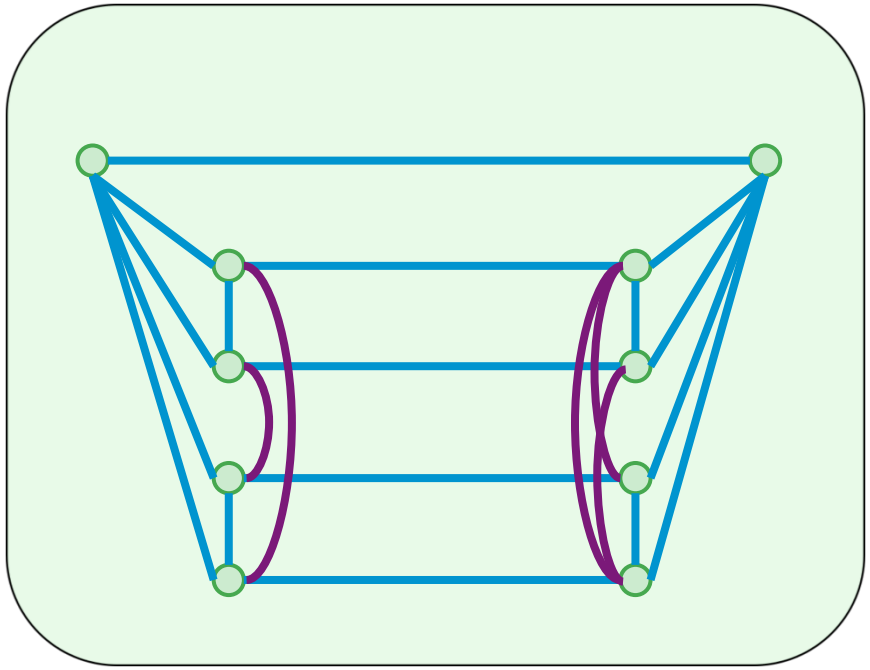

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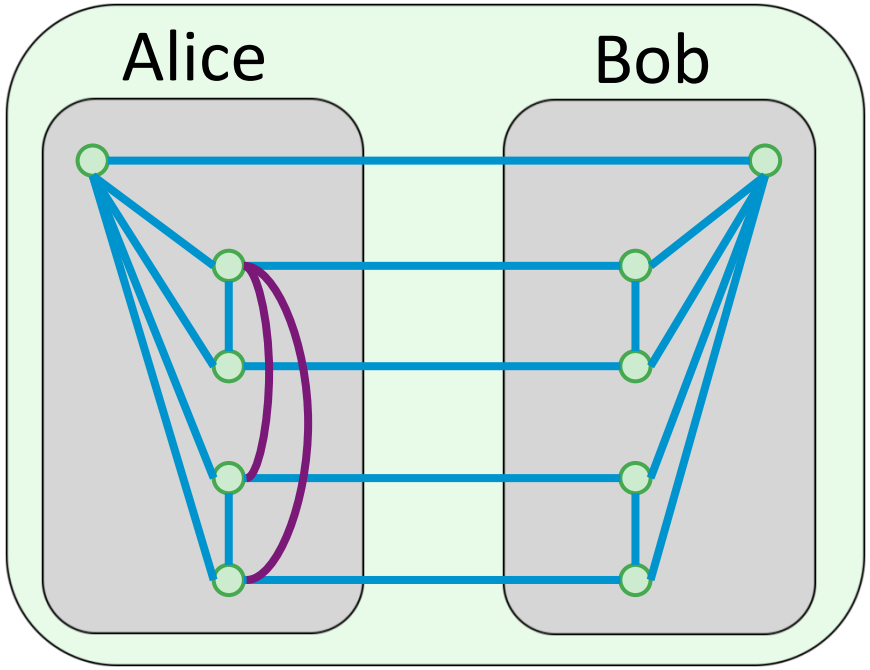


D=2 or **D=3** ?

Alice: $(a_1,\ldots,a_k)$ in $\{0,1\}^k$

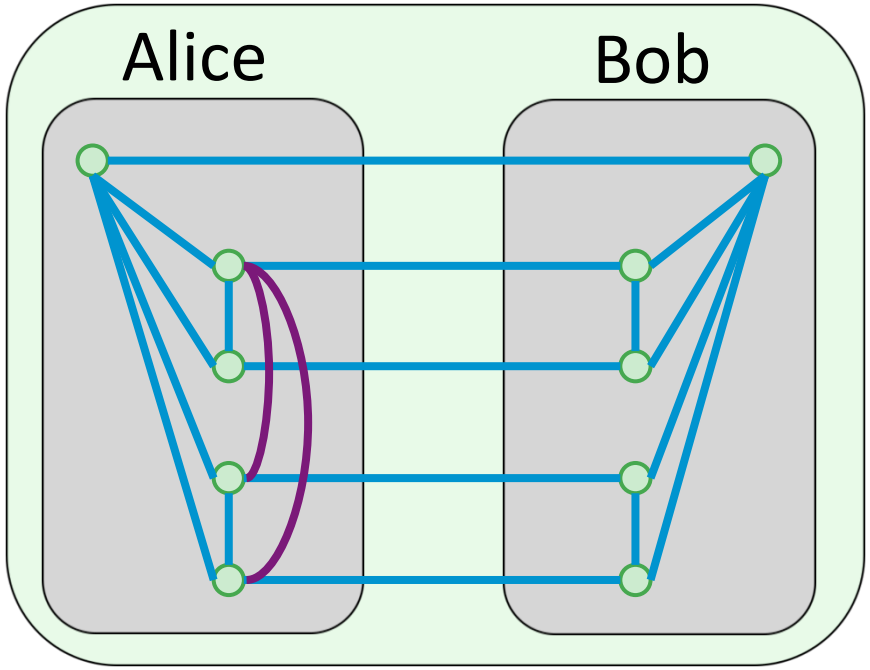
Bob: $(b_1,\ldots,b_k)$ in $\{0,1\}^k$

edge ⇔ input is 0

D=2 ⇔ inputs disjoint

Computing D

$\Omega(n/\log n)$ lower bound for deciding **D=2** or **D=3**
[Frischknecht, Holzer, Wattenhofer 2012]



D=2 or **D=3** ?

set-disjointness: **$\Omega(k)$** bits

Here: **$k=\Theta(n^2)$**

Size of cut: **$\Theta(n)$**

Diameter: **$\Omega(k/n\log n)=\Omega(n/\log n)$** rounds

Approximating D

$\Omega(n^{1/2}/\log n)$ for (3/2-ε)-approximation of D
[Frischknecht, Holzer, Wattenhofer 2012]

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for 3/2-approximation of D
$O(n^{3/4}+D)$ [Holzer, Wattenhofer 2012]
$O(Dn^{1/2}\log n)$ [Peleg, Roditty, Tal 2012]
$O((n)^{1/2}\log n+D)$ [Lenzen, Peleg 2013]
$O((n/\log n)^{1/2}+D)$
[Holzer, Peleg, Roditty, Wattenhofer 2014]

(3/2-ε)-approximation of D

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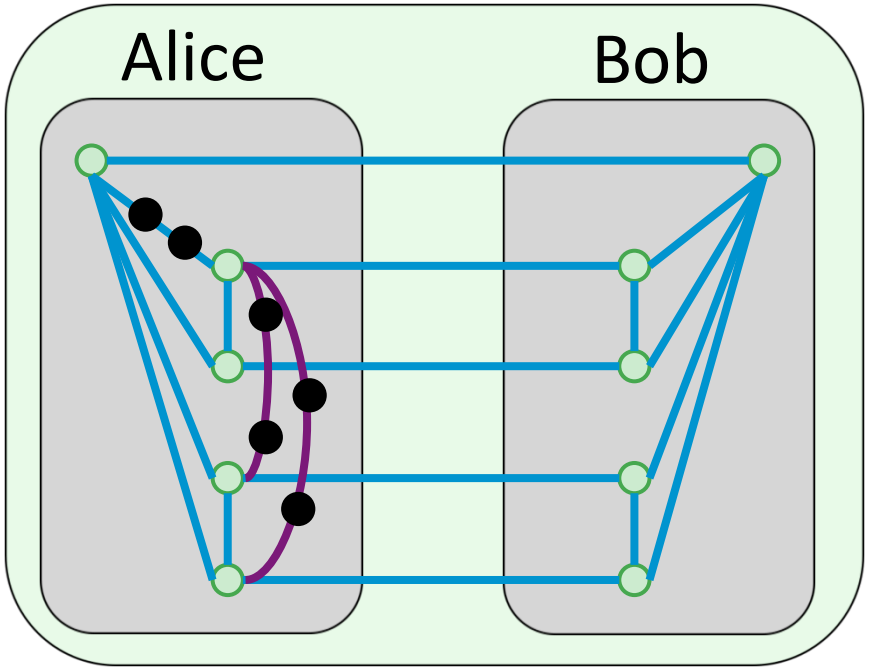

Add nodes on edges

(3/2-ε)-approximation of D

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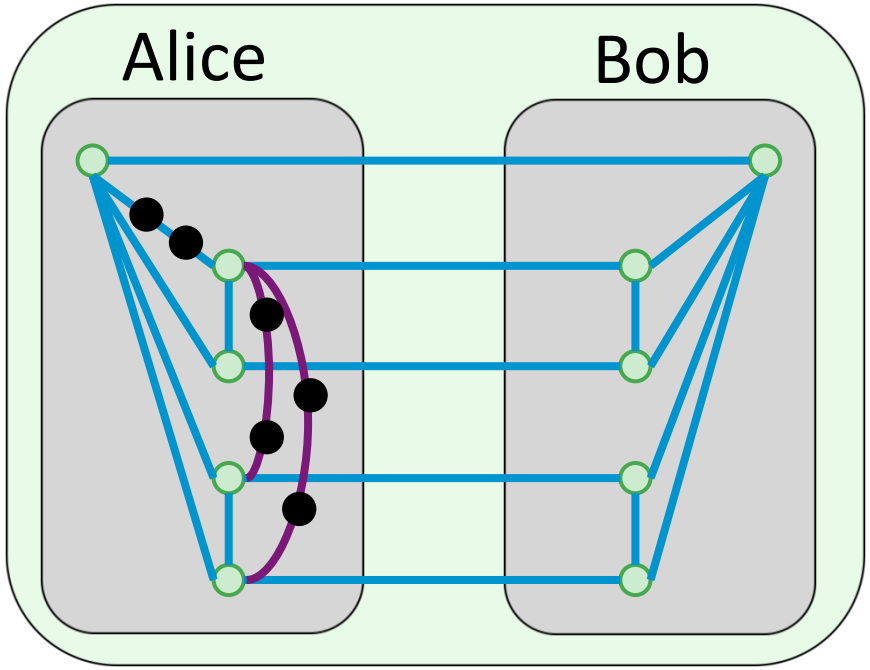

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New Distance Computation Bounds

$\Omega(n/\log^3 n)$ for (3/2-ε)-approximation of **D**
[Abboud, C-H., Khoury 2016]

sparse construction with a **smaller cut**

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sparse construction with a **smaller cut**
can be made **constant degree**

near-linear lower bounds for additional problems

- Computing the radius
- Approximating all eccentricities
- Verifying spanners

New Distance Computation Bounds

$\Omega(n/\log^2 n)$ for computing **D**

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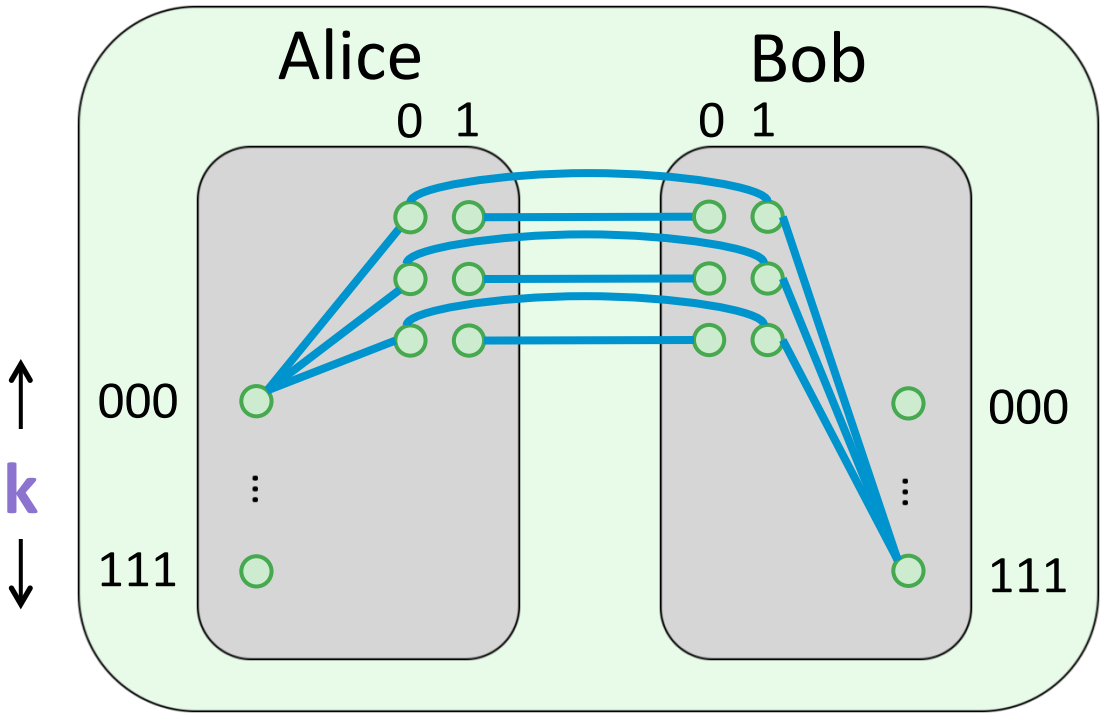

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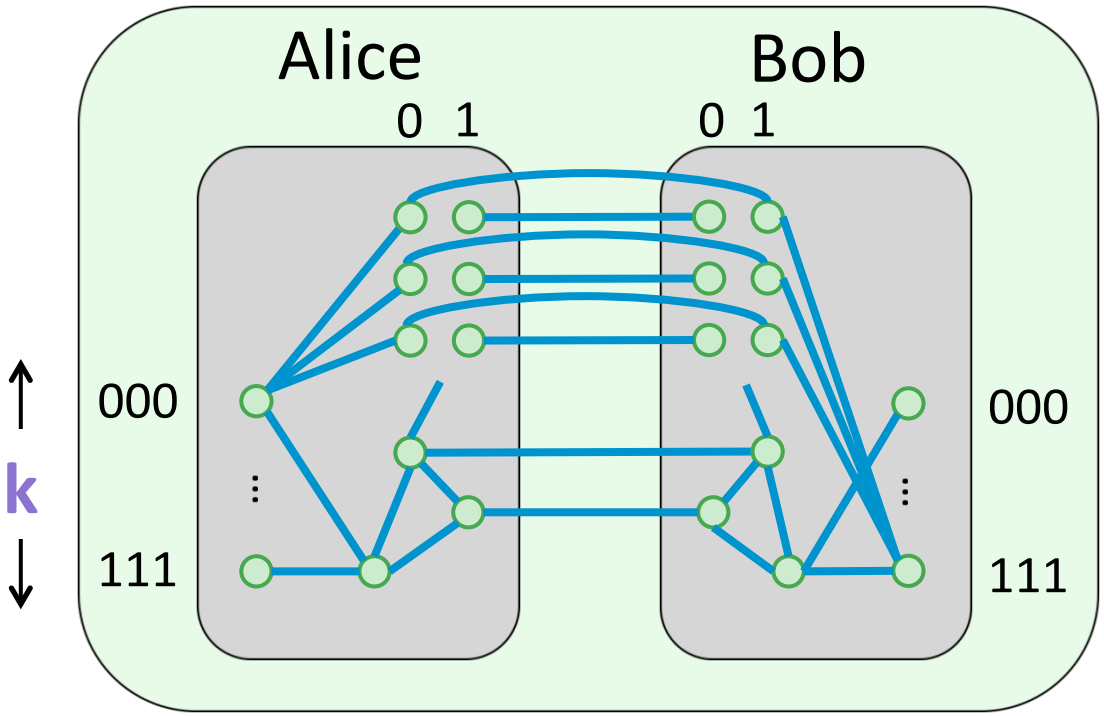

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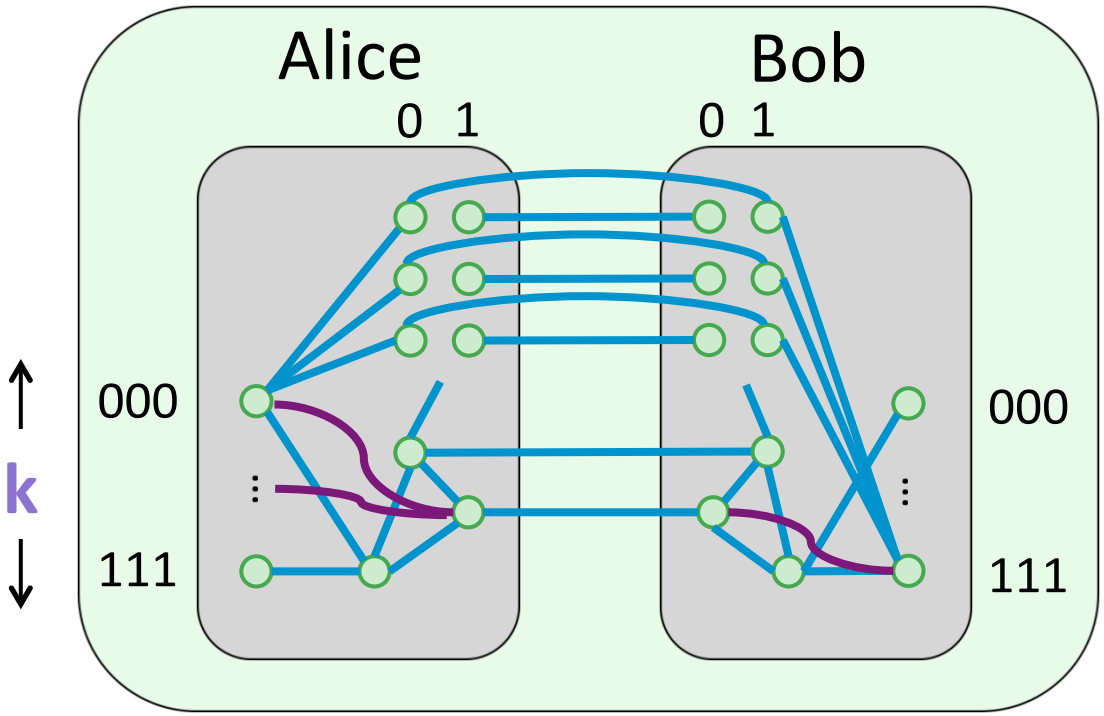

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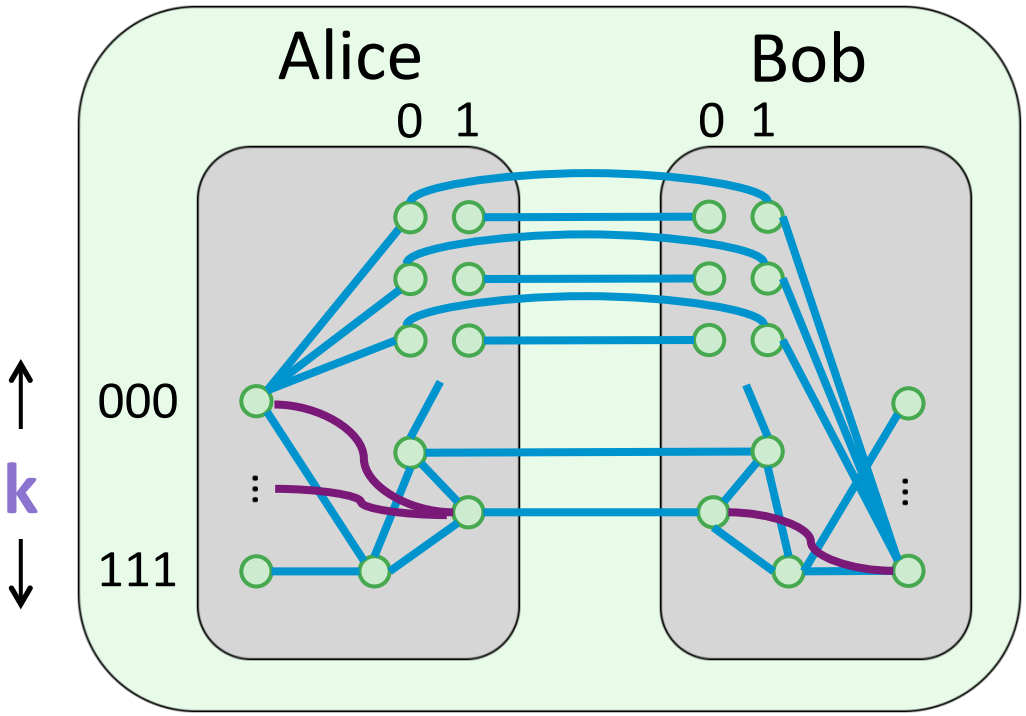

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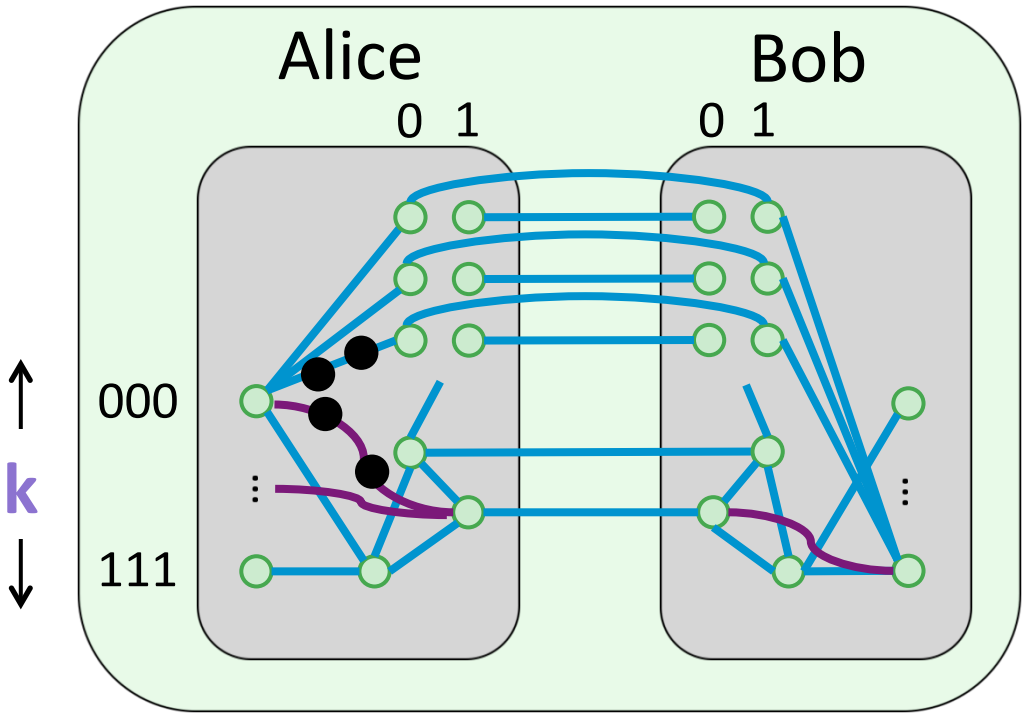

Add nodes on edges

CONGEST – Computing MST

MST = Minimum Spanning Tree

$O(n\log n)$	[Gallager, Humblet, Spira 1983]
$O(n)$	[Awerbuch 1987]
$O(D+n^{0.613}\log^* n)$	[Garay, Kutten, Peleg 1998]
$O(D+n^{1/2}\log^* n)$	[Kutten, Peleg 1998]

CONGEST – Computing MST

MST = Minimum Spanning Tree

$\Omega(n^{1/2}/\log n)$ for $D=\Omega(\log n)$ [Peleg, Rubinovich 1997]
$\Omega(n^{1/3}/\log^{1/2} n)$ for $D=4$,
$\Omega(n^{1/4}/\log^{1/2} n)$ for $D=3$ [Lotker, Patt-Shamir, Peleg 2001]
$\Omega((n/\alpha \log n)^{1/2})$ α-approx [Elkin 2004]
$\Omega(n^{1/2}/\log^{1/2} n)$ [Das-Sarma, Holzer, Kor, Korman, Nanongkai, Pandurangan, Peleg, Wattenhofer 2011]

Computing MST

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Reduction from subgraph connectivity:

$w(e)=0 \Leftrightarrow$ e is in subgraph H
$w(MST)=0 \Leftrightarrow$ H is connected

Subgraph Connectivity

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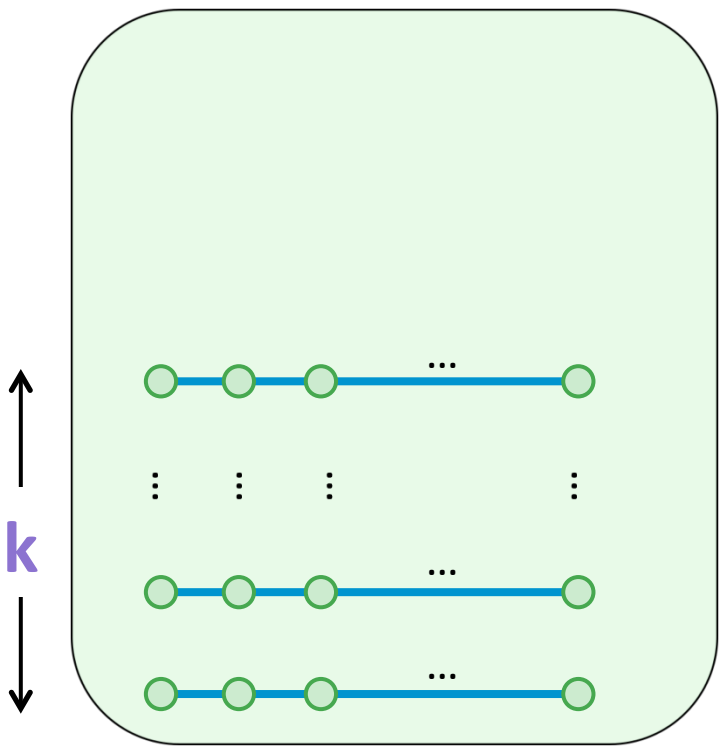

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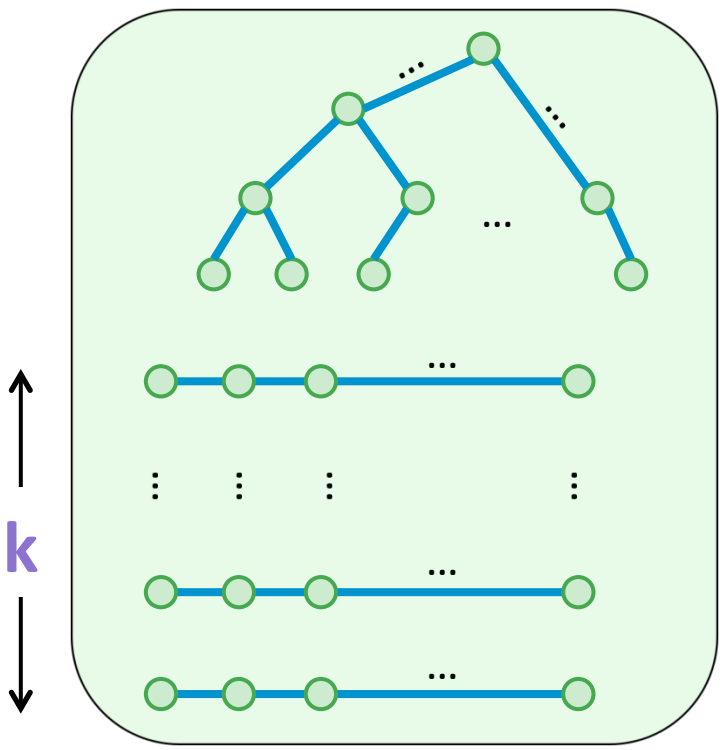

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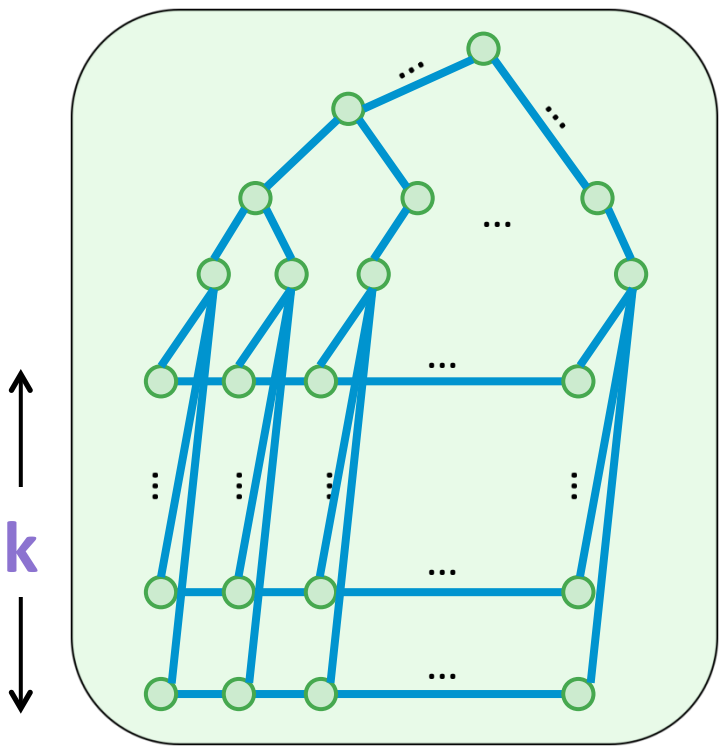

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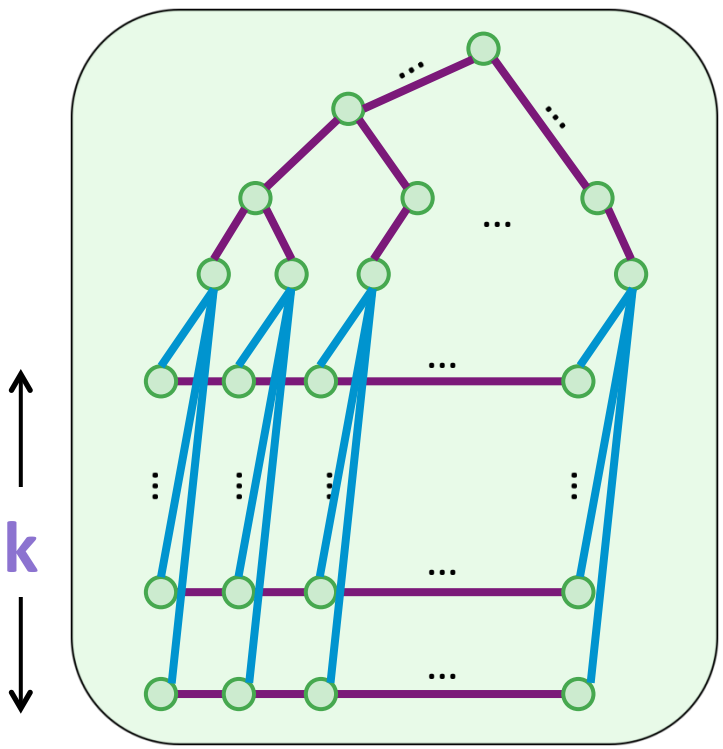

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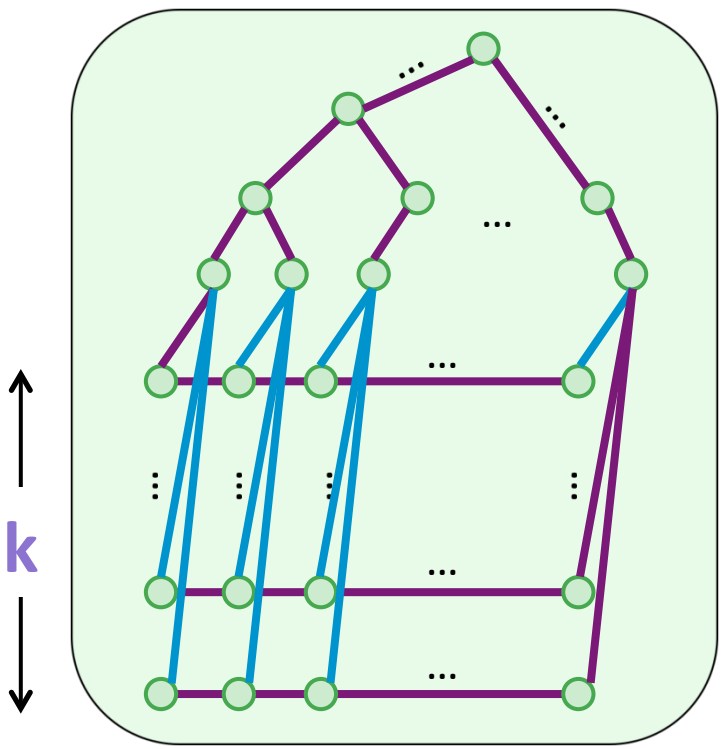

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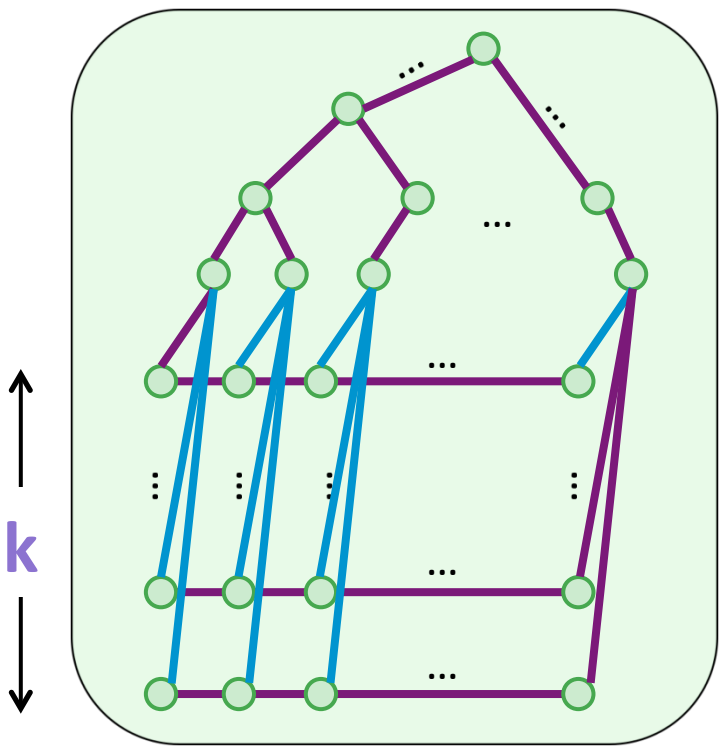

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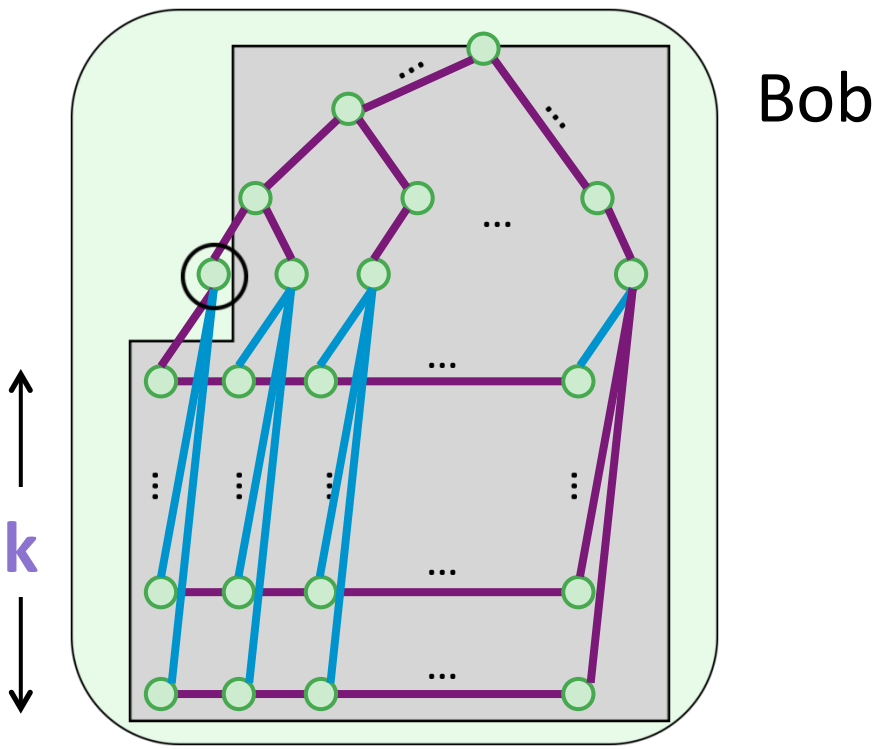

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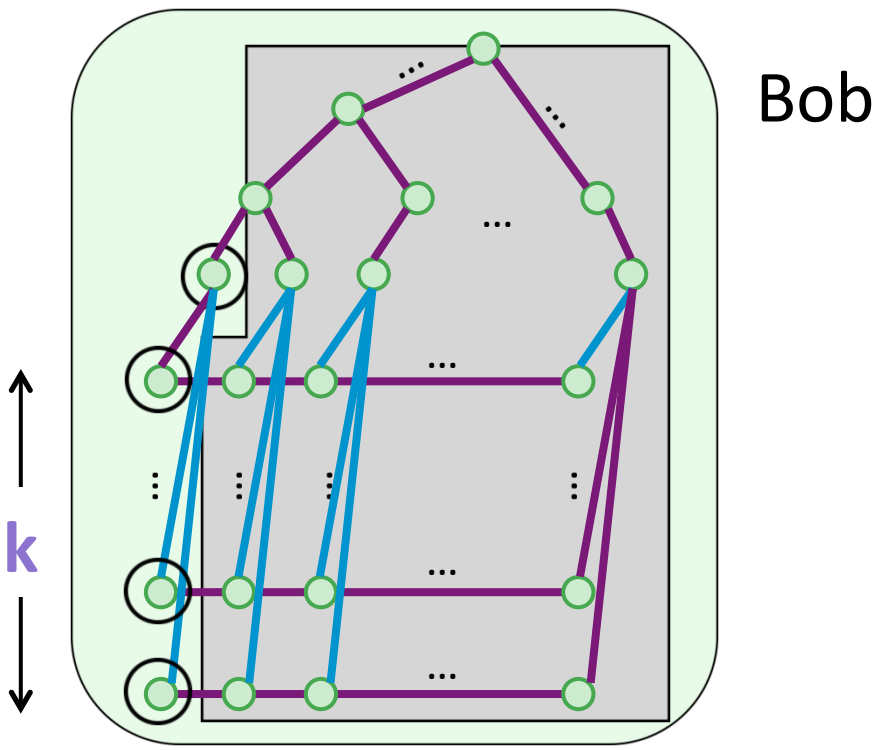

set-disjointness: $\Omega(k)$ bits

Here: $k=\Theta(n^{1/2})$

Subgraph Connectivity

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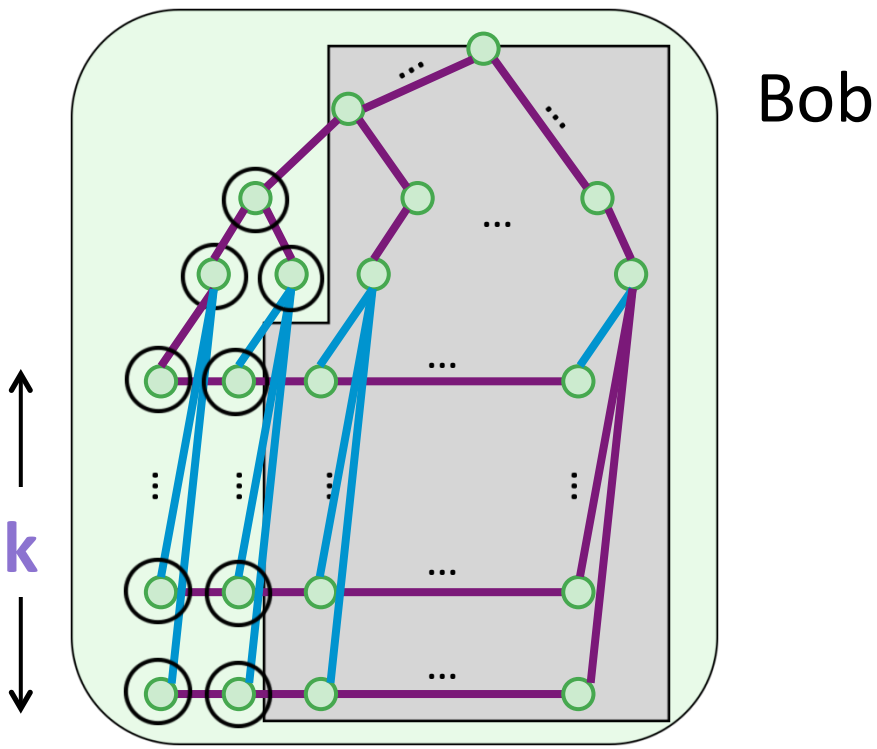

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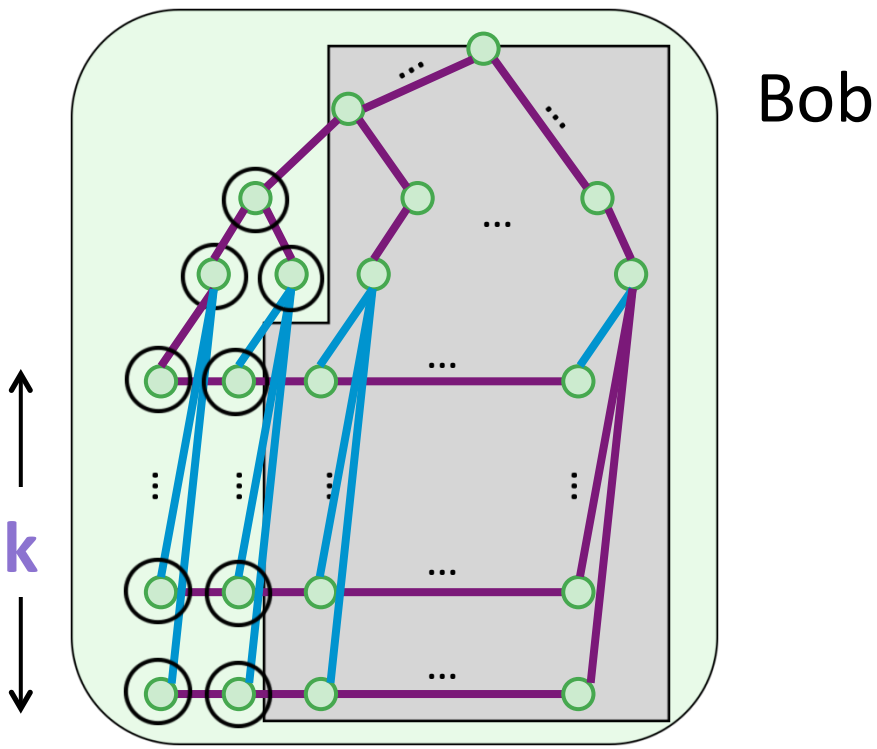

set-disjointness: $\Omega(k)$ bits

Here: $k=\Theta(n^{1/2})$

Used cut: $\Theta(\log n)$

Connected subgraph: $\Omega(n^{1/2}/\log^2 n)$ rounds

Spanners

S is an **(α,β)-spanner** of G if for u,v in V:

$$d_S(u,v) \leq \alpha d_G(u,v) + \beta$$

CONGEST – Spanners

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CONGEST spanners

[Baswana, Sen 2003, Baswana, Kavitha, Melhorn, Pettie 2005, Pettie 2008]

Ω(D) for various additive spanners [Pettie 2008]

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NEW: **Ω(|P|/nlogn)** for **(+2)-Pairwise** spanners
[C-H., Kavitha, Paz, Yehudayoff 2016]

New Spanner Bounds

$\Omega(|P|/n\log n)$ for **(+2)-Pairwise** spanners
[C-H., Kavitha, Paz, Yehudayoff 2016]

Need stretch **+2** for a set **P** of pairs in **VxV**

For **$|P|=n^{3/2}$** the bound is **Ω-tilde($n^{1/2}$)**

New Spanner Bounds

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[C-H., Kavitha, Paz, Yehudayoff 2016]

partial-complement(**p**,**m**):
Alice: set **x** of **p** elements in **{1,…,m}**
Bob: set **y** of **m/2** elements in **{1,…,m}**, not in **x**

partial-complement(**p**,**m**) requires **$\Omega(p)$** bits

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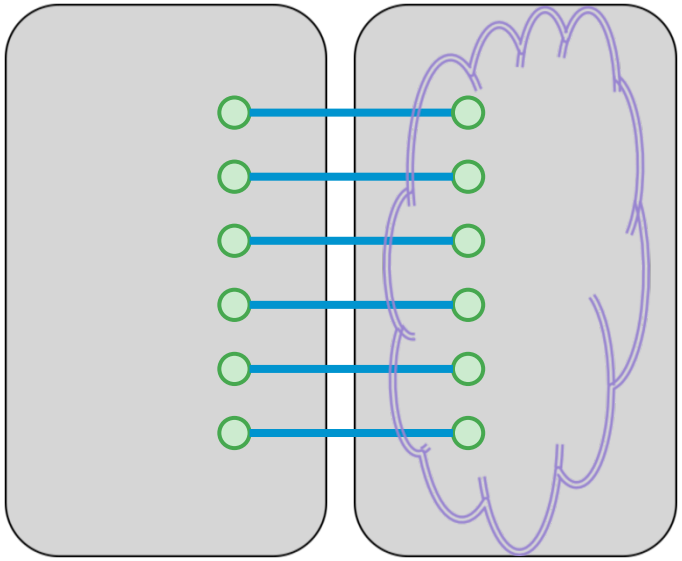
Alice



Bob

$|P|=\Theta(n^{3/2})$

Number of edges: **$\Theta(n^{3/2})$**
Girth (smallest cycle): **6**
Diameter: **3**

Must remove
$\Theta(n^{3/2})$ edges

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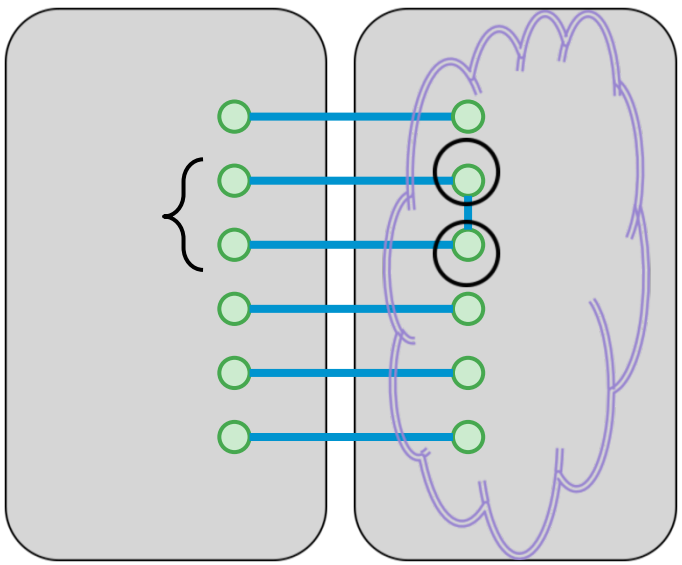
$d_G(u,v)=3$



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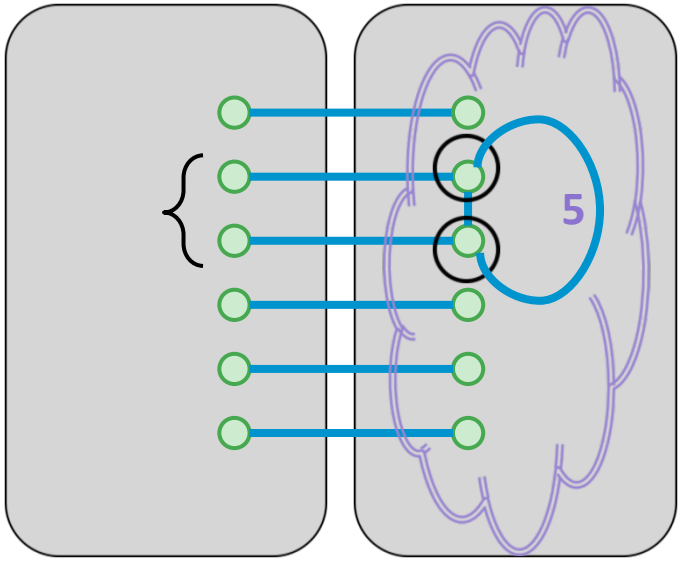
$d_S(u,v)=7$



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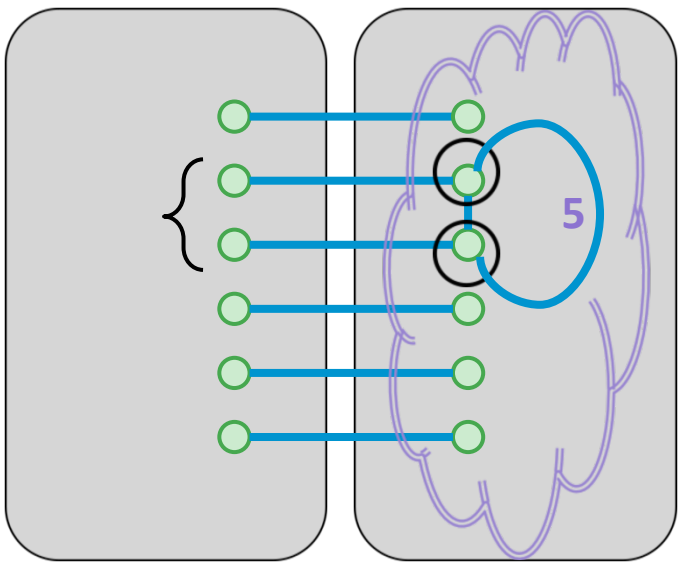

Bob

Number of edges: $\Theta(n^{3/2})$
Girth (smallest cycle): **6**
Diameter: **3**

solves partial-complement($p=|P|$, $m=n^{3/2}$)

CONGEST – Additional Lower Bounds

Detecting small subgraphs [Drucker, Kuhn, Oshman 2014]

Random walks [Nanongkai, Das-Sarma, Pandurangan 2014]

Small cuts [Ghaffari, Kuhn 2013,
Das-Sarma, Molla, Pandurangan 2015]

Broadcast congested clique [Drucker, Kuhn, Oshman 2014,
Holzer, Pinsker, 2015]

Message lower bounds [Pandurangan, Peleg, Scquizzato 2016]

k-machine model [Pandurangan, Robinson, Scquizzato 2016]

What's next?

Specific open problems

- In the various models

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- In the various models

Planar graphs

- MST in **O(DlogD)** [Ghaffari, Haeupler 2016]
- Distance computation?
  - Our constructions: far from planar

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Thank you!

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